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Original Article

Some Ridge Biasing Parameter for Linear Regression Model and Their Performances on Kibria-Lukman Estimator

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ABSTRACT

Multicollinearity, arising from the violation of the independence assumption among explanatory variables in a linear regression model, poses a significant challenge to parameter estimation. It inflates the variances of the Ordinary Least Squares (OLS) estimates, leading to unstable coefficient estimates and unreliable inference. To mitigate this problem, several biased estimators such as the Ridge and Liu estimators have been developed. Recently, Kibria and Lukman (2020) introduced the Kibria–Lukman Estimator (KLE), a ridge-type alternative designed to improve estimation accuracy under multicollinearity. However, the efficiency of ridge-type estimators critically depends on the choice of the biasing parameter, which controls the trade-off between bias and variance. This study conducts a comprehensive evaluation of 25 existing ridge biasing parameters alongside three newly proposed parameters within the KLE framework. The proposed estimators were assessed using extensive Monte Carlo simulations under varying levels of multicollinearity and sample sizes. Performance was evaluated based on the Mean Squared Error (MSE) criterion. The results reveal that the proposed estimator, Ridge_kgk, consistently outperforms other competing estimators, demonstrating superior efficiency and stability across different data conditions. The findings highlight the potential of the new biasing parameters in enhancing the robustness and predictive accuracy of ridge-type estimators in multicollinearity regression settings.

Keywords: Regression, Multicollinearity, Kibra –Lukman estimator, Simulation study, Mean Square Error

INTRODUCTION

Multiple Linear Regression (MLR) extends the simple linear regression framework by incorporating two or more explanatory variables into a single predictive model for a continuous response variable. The general form of the model is expressed as follows: $Y_i = \beta_1 + \beta_2 x_{2i} + \beta_3 x_{3i} + \cdots + \beta_k x_{ki} + \varepsilon_i$ (1)

For i $=1,\cdots,M$, β_s are regression coefficients, X_{ji}, j = 2, 3, $\cdots$, k are the independent variables, Y_i is the dependent variable and ε_i is the stochastic error term. In matrix form, the M equations can be written as:

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_M \end{bmatrix} = \begin{bmatrix} 1 & x_{21} & \cdots & x_{k1} \\ 1 & x_{21} & \cdots & x_{k2} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{2M} & \cdots & x_{KM} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_M \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_M \end{bmatrix} = X\beta + \varepsilon$$

(2)

Where y denotes an n × 1 vector of observed response, β represents a p × 1 vector of unknown regression

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coefficients, X is an n × p matrix of observed explanatory variables and e is an n × 1 vector of random error terms assumed to follow a multivariate normal distribution with mean vector 0 and covariance matrix $\sigma^2 I_n$, where I_n is an identity matrix of order n. The Ordinary Least Square (OLS) estimator of β is therefore expressed as:

$$\hat{\beta} = (X'X)^{-1}X'y \qquad (3)$$

The covariance matrix of $\hat{\beta}$ is estimated as Cov ($\hat{\beta}$) $=\sigma^2(X'X)^{-1}$. It is evident that both the estimator $\hat{\beta}$ and its covariance structure are highly dependent on the properties of the matrix $X'X$.

1.1 Ridge Regression

The Ridge regression (RR), originally introduced in 1970 by Hoerl and Kennard, was developed to address the issue of multicollinearity commonly encountered in engineering and other empirical data analyses. Their pioneering study revealed that the introduction of a positive ridge parameter k leads to a ridge regression estimator whose Mean Squared Error (MSE) is lower than the variance of the Ordinary Least Squares (OLS) estimator, thereby achieving greater estimation efficiency through an optimal bias–variance trade-off. Consequently, the ridge regression estimator (RRE) is defined as follow:

$$\hat{\beta}(\mathrm{k}) = (Z + \mathrm{k}I_p))^{-1}X'y = \mathrm{W}\hat{\beta} \qquad (4)$$

Where M= $[I_p + kZ^{-1}]^{-1}$,k ≥ 0, Z=$X'X$, and I_p denotes an identity matrix of order p, This expression defines the ridge regression estimator. Since the matrix [Z+ k I_p] remains invertible for all k > 0, a unique solution for $\hat{\beta}(k)$. The ridge estimator is inherently biased; however, for a positive ridge parameter k, it often achieves a smaller MSE than the OLS estimator. From equ. (4), it follows that as $k \to 0$, $\hat{\beta}$(k)→ ∞, $\hat{\beta}$(k)→ 0. The parameter k often referred to as the ridge or biasing parameter, must be estimated from empirical data. In recent decades, considerable research efforts in the domains of multicollinearity and ridge regression estimation have focused on determining appropriate and efficient methods for estimating k. Numerous scholars have contributed to this line of inquiry, proposing various modified forms of ridge-type estimators. Notably, Hoerl and Kennard (1970) introduced the original ridge regression estimator, which was subsequently extended through the development of the Modified Ridge Regression (MRR) estimator, the Liu estimator (Liu, 1993), and more recently, the Kibria–Lukman estimator (Kibria and Lukman, 2020).

1.2 The Kibria Lukman Estimator

The newly formulated one-parameter estimator is obtained by optimizing the following objective function, designed to balance bias and variance in the estimation process:

$$(y - X\beta)'(y - X\beta) + k\left[(\beta + \hat{\beta})'(\beta + \hat{\beta}) - c\right] \qquad (5)$$

Minimization of the objective function with respect to β leads to the corresponding normal equations.

$$(X'X + KI_P)\beta = X'y - k\hat{\beta} \qquad (6)$$

In this formulation, k represents a nonnegative constant. Solving the preceding equation with respect to β produces the explicit form of the proposed estimator as:

$$\hat{\beta}_{KL} = (Z + kI_P)^{-1}(Z - kI_P)\hat{\beta} = E(K)F(K)\hat{\beta} \qquad (7)$$

Where $Z = X'X, E(K) = [I_P + kZ^{-1}]^{-1}$ and $F(K) = [I_P - kZ^{-1}]$. The proposed estimator, hereafter referred to as the Kibria–Lukman (KL) estimator, is denoted by$\hat{\beta}_{KL}$ and serves as a ridge-type modification of the conventional OLS estimator, and the biasing parameter k ≥ (Kibria and Lukman 2020). As with any regression estimator, the determination of an appropriate biasing parameter in the recently developed Kibria–Lukman Estimator (KLE) is crucial for assessing its efficiency and overall performance. Over the years, several studies have proposed and examined various estimators for the ridge regression biasing parameter (k). Foundational contributions include those of Hoerl and Kennard (1970), Hoerl, Kennard, and Baldwin (1975), McDonald and Galarneau (1975), Lawless and Wang (1976), and Dempster, Schatzoff, and Wermuth (1977). Subsequent advancements were made by Gibbons (1981), Kibria (2003), Khalaf and Shukur (2005), Alkhamisi and Shukur (2008), Muniz and Kibria (2009), Muniz, Kibria, Mansson, and Shukur (2012), and Mansson, Shukur, and Kibria (2010). More recent developments include the works of Hefnawy and Farag (2013), Aslam (2014), and Arashi and Valizadeh (2015), Durogade (2016), Lukman and Ayinde (2017), Owolabi *et al* (2022), Kibra (2022), Adedoyin *et al* (2025) among others. Despite these extensive efforts, there has been limited discussion regarding the interplay between multicollinearity and the error variance (noise parameter) a situation in which a high degree of multicollinearity is often accompanied by inflated error variance. This challenge can substantially affect the performance of existing biasing parameter estimators. Therefore, in this study, we propose a new estimator for the biasing parameter (k) within the Kibria–Lukman Estimator (KLE) framework to effectively address this limitation.

1 Statistical Methodology

2.1 Canonical Form

The canonical form of the model is:

$$y = A\alpha + \varepsilon \qquad (8)$$

Where A = XP and α= $P'\beta$. Here, P is an orthogonal matrix such that

$A'A = PX'XP = \Lambda = diag(\lambda_1, \lambda_2, \dots, \lambda_p)$. The OLS estimator of α is:

$$\hat{\alpha} = \Lambda^{-1}A'y, \qquad (9)$$

$$MSEM(\hat{\alpha}) = \sigma^2\Lambda^{-1} \qquad (10)$$

The ridge estimator (RE) of α is:

$$\hat{\alpha}(k) = J(K)\hat{\alpha} \qquad (11)$$

Where $J(k) = [I_P + k\Lambda^{-1}]^{-1}$ and k is the biasing parameter

$$MSEM(\hat{\alpha}(k)) = \sigma^2 J(k)\Lambda^{-1}J(k) + (J(k) - I_p)\alpha\alpha(J(k) - I_p)', \qquad (12)$$

Where $(J(k) - I_P) = -k(\Lambda + kI_P)^{-1}$

Thus the MSE of the propose estimator can be written as:

$$MSEM(\hat{\beta}_{KL}) = \sigma^2 E(k)F(k)S^{-1}F'(k)F'(k) + [E(k)F(k) - I_P]\beta\beta'[E(k)F(k) - I_P]' \qquad (13)$$

Finally, the MSE of Kibria-Lukman estimator after using the above stated definitions can be written as:

$$P(k) = MSEM[\hat{\alpha}_{KL}] = tr[MSEM(\hat{\alpha}_{KL})] \qquad (14)$$

$$P(k) = \sigma^2 \sum_{i=1}^{p} \frac{(\lambda_i - k)^2}{\lambda_i(\lambda_i + k)^2} + 4k^2 \sum_{i=1}^{p} \frac{\alpha_i^2}{(\lambda_i + k)^2} \qquad (15)$$

Differentiating $P(k)$ with respect to k gives and setting $(\partial p(k)/\partial k) = 0$, we obtain

$$\hat{k} = \frac{\sigma^2}{2\alpha_i^2 + (\sigma^2/\lambda_i)} \qquad (16)$$

The optimal value of k in (16) depends on the unknown parameter σ^2 and α^2. These two estimators are replaced with unbiased estimate.

2.2 Biasing Parameters

In accordance with the methodology introduced by Hoerl, Kennard, and Baldwin, the harmonic mean formulation corresponding to Equation (16) is defined as:

$$\hat{k}_{hmn}^{LS} = \frac{P\hat{\sigma}^2}{\sum_{i=1}^{p}[2\alpha_i^2 + (\sigma^2/\lambda_i)]} \qquad (17)$$

As proposed by Özkale and Kaciranlar, the minimum form of equation (16) is expressed as:

$$\hat{k}_{min}^{LS} = min\left[\frac{\hat{\sigma}^2}{2\alpha_i^2 + (\sigma^2/\lambda_i)}\right] \qquad (18)$$

Where the λ_i are eigenvalues of the matrix X′X, $\hat{\alpha}_i$ is the i[th] element of $\hat{\alpha}$, and

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{p}\hat{e}_i^2}{n-p}$$

$$\hat{e}_i = y_i - X_j'\hat{\alpha}_i$$

We next examine the existing methods proposed in the literature for determining the value of k. Hoerl and Kennard (1970) recommended estimating k as (here denoted by $\hat{k}_{HK}$)

$$\hat{k}_{HK} = \frac{\hat{\sigma}^2}{\hat{\alpha}_{max}^2} \qquad (19)$$

Here, $\hat{\alpha}_{max}^2$represents the largest element of $\hat{\alpha}_i$. Hoerl and Kennard (1970) demonstrated that the estimator $\hat{k}_{HK}$ yields a smaller Mean Squared Error (MSE) compared to the Ordinary Least Squares (OLS).

Hoerl *et al*. (1975) defined the ridge parameter k (denoted here by $\hat{k}_{HKB}$)

$$\hat{k}_{HKB} = \frac{P\hat{\sigma}^2}{\hat{\alpha}'\hat{\alpha}} \qquad (20)$$

Lawless and Wang (1976) derived biasing parameter k to be (denoted here by $\hat{k}_{LW}$)

$$\hat{k}_{LW} = \frac{P\sigma^2}{\hat{\alpha}'X'X\hat{\alpha}} \qquad (21)$$

Hocking, Speed and Lynn (1976) also derived shrinkage parameter k to be (denoted here by $\hat{k}_{HSL}$)

$$\hat{K}_{HSL} = \hat{\sigma}^2 \frac{\sum_{i=1}^{p}(\lambda_i\hat{\alpha}_i)^2}{(\sum_{i=1}^{p}\lambda_i\,\hat{\alpha}_i^2)^2} \qquad (22)$$

Kibria (2003) suggested alternative biasing estimators for k derived from the geometric mean (GM) and median of $\hat{\sigma}^2/\hat{\alpha}_i^2$. These estimators are expressed as follows:

$$\hat{k}_{GM} = \frac{\hat{\sigma}^2}{(\prod_{i=1}^{p}\hat{\alpha}_i^2)^{\frac{1}{p}}} \qquad (23)$$

$$\hat{k}_{MED} = median\left\{\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}\right\}, i = 1, 2, \dots, p \qquad (24)$$

Based on modification of $\hat{k}_{HK}$, Khalaf and Shukur (2005) suggested k to be (denoted by $\hat{k}_{KS}$)

$$\hat{k}_{KS} = \frac{\lambda_{max}\hat{\sigma}^2}{(n-p)\hat{\sigma}^2 + \lambda_{max}\hat{\alpha}_{max}^2} \qquad (25)$$

Where λ_{max} is the maximum eigen value of the matrix $X'X$. Building on the work of Kibria (2003) and Khalaf and Shukur (2005), Alkhamisi, Khalaf, and Shukur (2006) proposed the following three estimators for k $\hat{k}_{arith}^{KS} =$

$$\frac{1}{p}\sum_{i=1}^{p}\frac{\lambda_i\hat{\sigma}_i^2}{(n-p)\hat{\sigma}_i^2+\lambda_i\hat{\alpha}_i^2} \qquad (26)$$

$$\hat{k}_{max}^{KS} = \max\left(\frac{\lambda_i\hat{\sigma}_i^2}{(n-p)\hat{\sigma}_i^2 + \lambda_i\hat{\alpha}_i^2}\right) \qquad (27)$$

$$\hat{k}_{md}^{KS} = \text{median}\left(\frac{\lambda_i\hat{\sigma}_i^2}{(n-p)\hat{\sigma}_i^2 + \lambda_i\hat{\alpha}_i^2}\right) \qquad (28)$$

Drawing upon the geometric mean and square-root methods proposed by Khalaf and Shukur (2005), Kibria (2003), and Alkhamisi et al. (2006), Muniz and Kibria (2009) developed seven new estimators for the ridge parameter k:

$$\hat{k}_{gm}^{KS} = \prod_{i=1}^{p}\left(\frac{\lambda_i\hat{\sigma}_i^2}{(n-p)\hat{\sigma}_i^2 + \lambda_i\hat{\alpha}_i^2}\right)^{\frac{1}{p}} \qquad (29)$$

$$\hat{k}_{KM2} = \max\left(\frac{1}{\sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}}}\right) \qquad (30)$$

$$\hat{k}_{KM3} = max\left(\sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}}\right) \qquad (31)$$

$$\hat{k}_{KM4} = \left(\prod_{i=1}^{p}\frac{1}{\sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}}}\right)^{\frac{1}{p}} \qquad (32)$$

$$\hat{k}_{KM5} = \left(\prod_{i=1}^{p}\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}\right)^{\frac{1}{p}} \qquad (33)$$

$$\hat{k}_{KM6} = \text{median}\left(\frac{1}{\sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}}}\right) \qquad (34)$$

$$\hat{k}_{KM7} = median\left(\sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}}\right) \qquad (35)$$

Following the square-root transformation methodology of Alkhamisi and Shukur (2008), Muniz *et al.* (2012) introduced five new estimators for the ridge parameter *k*:

$$\hat{k}_{KM8} = \max\left(\frac{1}{q_i}\right) \qquad (36)$$

$$\hat{k}_{KM9} = max(q_i) \qquad (37)$$

$$\hat{k}_{KM10} = \left(\prod_{i=1}^{p}\frac{1}{q_i}\right)^{\frac{1}{p}} \qquad (38)$$

$$\hat{k}_{KM11} = \left(\prod_{i=1}^{p} q_i\right)^{\frac{1}{p}} \qquad (39)$$

$$\hat{k}_{KM12} = \text{median}\left(\frac{1}{q_i}\right) \qquad (40)$$

Where,

$$q_i = \frac{\lambda_{max}\hat{\sigma}^2}{(n-p)\hat{\sigma}^2 + \lambda_{max}\,\hat{\alpha}_i^2} \qquad (41)$$

Khalaf (2012), based on modification of $\hat{k}_{HK}$, proposed *k* to be (denoted by $\hat{k}_{GKK}$)

$$\hat{k}_{GKK} = \hat{k}_{HK} + \frac{2}{(\lambda_{max} + \lambda_{min})'} \qquad (42)$$

Where λ_{max} and λ_{min} are the maximum and minimum eigenvalues of matrix $X'X$ respectively.

Nomura (1988) proposed estimating the ridge parameter *k* as (denoted by $\hat{k}_{HMO}$)

$$\hat{k}_{HMO} = \frac{P\hat{\sigma}^2}{\sum_{I=1}^{P}\left[\hat{\alpha}_1^2 \Big/ 1 + \left[1 + \lambda_i\left(\frac{\hat{\alpha}_i^2}{\hat{\sigma}^2}\right)^{\frac{1}{p}}\right]\right]} \qquad (43)$$

3 The Proposed Biasing Estimator

Following the modification of Khalaf (2012), a new biasing parameter was proposed and defines as:

$$\hat{k}_{GK} = \hat{k}_{HK} + 2/\lambda_{max} + \lambda_{min} \qquad (44)$$

Following the work of ozkale and kaciranlar (2007), the maximum version and the median of (16) is proposed and define as:

$$\hat{k}_{max}^{LS} = max\left[\frac{\hat{\sigma}^2}{2\alpha_i^2 + (\sigma^2/\lambda_i)}\right] \qquad (45)$$

$$\hat{k}_{med}^{LS} = median\left[\frac{\hat{\sigma}^2}{2\alpha_i^2 + (\sigma^2/\lambda_i)}\right] \qquad (46)$$

Where λ_{max} and λ_{min} are the maximum and minimum eigenvalues of $X'X$ respectively.

4. Simulation Study

The primary objective of this study is to evaluate and compare the performance of various ridge biasing parameter estimators, with the aim of recommending efficient and reliable options for practical applications. Since a purely theoretical comparison among these estimators is not feasible, a simulation study was conducted using R software. The design of the simulation experiment was structured around factors that are expected to influence the statistical properties of the estimators under consideration, as well as the evaluation criteria employed to assess their performance. Given that the degree of multicollinearity among the explanatory variables (X's) plays a critical role in the behavior of ridge-type estimators, we adopted the data generation approach proposed by Kibria and Lukman (2020), Oladapo *et al* (2022,2023 and 2024), Idowu *et al* (2022 and 2023) and Owolabi *et al* (2022) where the explanatory variables were simulated using the following relationship:

$$X_{ij} = (1-\gamma^2)^{\frac{1}{2}} z_{ij} + \gamma z_{ip}\ , \quad i = 1,2,\dots,n, \quad j = 1,2,\dots,p \qquad (47)$$

Where z_{ij} represent independent standard normal pseudo-random numbers, and let γ denote the correlation between any two explanatory variables X, with values γ = 0.80, 0.90, 0.95, 0.99 for p = 5. These variables are standardized such that $X'X$ and $X'y$ are expressed in correlation forms. The n observations of y are generated according to the following equation:

$$y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{I2} + \ldots + \beta_p X_{ip} + e_i, \quad i = 1,2,\ldots,n \qquad (48)$$

Where the e_i ~NIID (0, σ^2). And $\beta'\beta = 1$ as in Lukman *et al*. (2021). The simulation was conducted with 5,000 replications, considering sample sizes of n=50 and n=100, and error standard deviations of σ =3.0, 5.0 and 10. In this table, the average values of k for the Kibria–Lukman estimators are reported, and the proportion of replications in which the KLS estimators yield a smaller Mean Squared Error (MSE) than the OLS estimator is indicated in parentheses. For comparison purposes, the estimated MSE is computed as follows:

$$MSE(\hat{\beta}) = \frac{1}{5000}\sum_{i=1}^{5000}(\hat{\beta}-\beta)'(\hat{\beta}-\beta) \qquad (49)$$

RESULT

This section reports the findings of the Monte Carlo simulation, focusing on the comparative performance of different biasing parameters against the Ordinary Least Squares (OLS) estimator in terms of Mean Squared Error (MSE). The key outcomes are illustrated through graphical analyses, providing a visual summary of estimator efficiency. Furthermore, detailed quantitative results for the five most efficient biasing parameters are presented in Tables 1 and 2, respectively.

Table 1: MSE Comparison With Various Biasing Parameter When n=50

n	σ /γ	0.8		0.9		0.95		0.99	
50	3	OLS	3.119261	OLS	6.396138	OLS	13.03209	OLS	72.35728
		ridgekgk	0.506875	ridgekgk	0.841694	ridgekgk	1.633292	ridgekgk	9.289061
		ridgekm9	0.900381	ridgekm9	1.334978	ridgekm5	3.296332	ridgeks	20.44684
		kls_max	1.019445	ridgekm5	1.951883	ridgekm7	3.346671	ridgekhmo	21.02746
		ridgekm5	1.233454	ridgekhmo	1.974866	ridgekm9	3.494403	ridgekm5	23.91627
		ridgekhmo	1.245669	ridgekm7	2.048751	ridgeksmx	3.577529	ridgekm7	25.02527
	5	0.8		0.9		0.95		0.99	
		OLS	8.744206	OLS	17.64085	OLS	36.5453	OLS	211.1493
		ridgekgk	1.065551	ridgekgk	2.04342	ridgekgk	4.304672	ridgekgk	25.99921
		ridgekm9	2.30903	ridgekm9	2.988793	ridgeksmx	8.997492	ridgeks	57.26265
		ridgekhmo	2.607262	ridgeksmx	3.992631	ridgekm5	9.006038	ridgekhmo	62.22484
		kls_max	2.943057	ridgekm5	4.783466	ridgekm9	9.006122	ridgekm4	71.38203
		ridgekm3	3.146505	ridgekhmo	4.921552	ridgekm7	9.493686	kls_med	71.83467
	10	0.8		0.9		0.95		0.99	
		OLS	34.99279	OLS	71.52686	OLS	147.5813	OLS	802.4952
		ridgekgk	3.775002	ridgekgk	7.9108	ridgekgk	17.21413	ridgekgk	101.2598
		ridgekm9	7.942291	ridgekm9	10.54621	ridgeksmx	35.78133	ridgeks	226.0707
		ridgekhmo	9.244573	ridgeksmx	12.16536	ridgekm9	36.01294	ridgekhmo	234.7825
		ridgeksmx	9.46138	ridgekm5	18.64363	ridgekm5	36.87991	ridgekm4	254.3872
		ridgekm5	11.12184	ridgekhmo	18.89106	ridgeks	38.13841	ridgekm11	280.2993

Table 2: MSE Comparison With Various Biasing Parameter When n=100

n	σ /γ	0.8		0.9		0.95		0.99	
100	3	OLS	1.456806	OLS	2.979783	OLS	6.136638	OLS	34.60792
		ridgekgk	0.333625	ridgekgk	0.461724	ridgekgk	0.819828	ridgekgk	4.445529
		kls_max	0.562144	ridgekm9	0.84869	ridgekm9	1.185489	ridgekm6	8.503839
		ridgekm9	0.725821	kls_max	0.943758	ridgekm5	1.797427	ridgekm4	8.539522
		ridgekm3	0.757978	ridgekhmo	1.10419	kls_max	1.837256	ridgekm5	9.234336
		ridgekm5	0.76469	ridgekm5	1.169561	ridgekm7	1.891241	ridgekm7	9.790814

	5	0.8		0.9		0.95		0.99	
		OLS	4.005733	OLS	8.42795	OLS	17.53722	OLS	93.26741
		ridgekgk	0.595486	ridgekgk	1.029015	ridgekgk	2.108074	ridgekgk	12.02214
		kls_max	1.241737	ridgekm9	2.168397	ridgekm9	3.055222	ridgeks	21.83337
		ridgekhmo	1.451163	ridgekhmo	2.424392	ridgeksmx	4.064918	ridgekm4	23.5622
		ridgekm3	1.652956	kls_max	2.653076	ridgekm5	4.806325	ridgekm6	24.11049
		kls_med	1.828434	ridgekm3	2.890744	ridgekm7	4.959914	ridgekm5	26.70134
	10	0.8		0.9		0.95		0.99	
		OLS	16.13528	OLS	32.77341	OLS	68.87816	OLS	374.3429
		ridgekgk	1.759601	ridgekgk	3.63438	ridgekgk	8.077204	ridgekgk	47.86255
		ridgekhmo	4.277563	ridgekm9	7.48795	ridgekm9	10.74171	ridgeks	69.34554
		ridgekm3	5.367357	ridgekhmo	8.538107	ridgeksmx	12.05585	ridgekm4	98.14063
		kls_max	5.783906	ridgeksmx	8.886951	ridgekhmo	18.04907	ridgekm6	99.82201
		kls_med	6.304446	ridgekm5	10.49928	ridgekm5	18.51706	ridgekhmo	106.8815

5.1 Performances with Respect To Sigma (σ)

Tables 1 and 2 present the Mean Squared Error (MSE) of the selected ridge biasing parameters as a function of σ, for sample sizes n=50,100 and correlation levels γ=0.8,0.9,0.95. The findings reveal that the MSE generally increases with higher values of σ. notably, all ridge-type estimators exhibit smaller MSEs compared to the Ordinary Least Squares (OLS) estimator, indicating improved estimation efficiency. Specifically, when σ=3, the Ridge_kgk estimator demonstrates superior performance relative to other biasing parameters in terms of lower MSE. A similar pattern is observed at σ=5 and σ=10 where Ridge_kgk consistently outperforms its counterparts. For clarity, Figure 1 illustrates the behavior of the estimators as a function of σ for γ=0.8 and n=50.

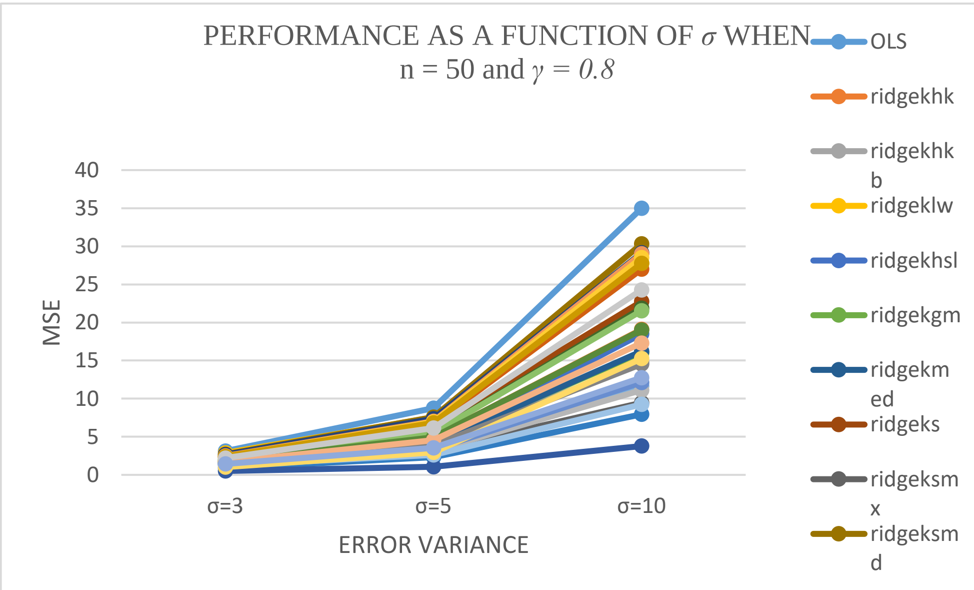

Figure 1: Performance as a function of sigma (σ) when *n = 50* and *γ = 0.8*

5.2 Performance with Respect To The Correlation Coefficient (γ)

The Mean Squared Errors (MSEs) of the selected estimators were further examined as a function of the correlation coefficient (γ) for given values of n, σ, and p. To enhance interpretability, the performance of the biasing parameters as a function of γ is depicted in Figure 2. The findings reveal that an increase in the correlation among explanatory variables leads to a corresponding rise in the MSE of ridge-type estimators. Nevertheless, all ridge estimators maintain smaller MSEs compared to the Ordinary Least Squares (OLS) estimator, confirming their efficiency in handling multicollinearity. For relatively low correlation levels (e.g., γ=0.8), the Ridge_kgk estimator exhibits superior performance with the smallest MSE. A similar dominance of Ridge_kgk is observed for γ=0.9 and γ=0.95, based on the minimum MSE criterion. Furthermore, even at a high correlation level (γ=0.99), Ridge_kgk continues to outperform other biasing parameters, demonstrating its robustness under severe multicollinearity.

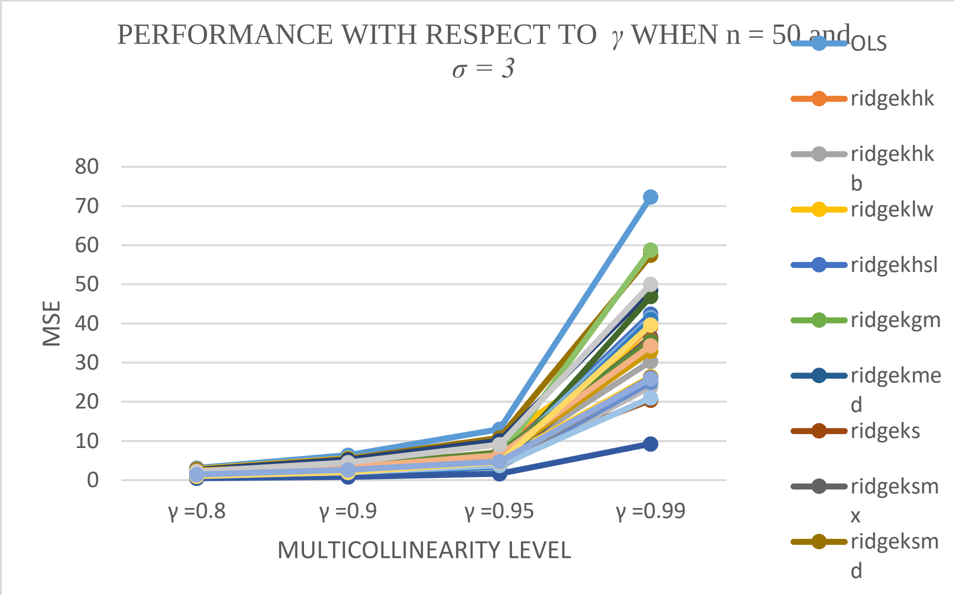

Figure 2: Performance as a function of correlation coefficient (*γ*) when *n=50* and *σ =3*

5.3 Performances with Respect To Sample Size (n)

The Mean Squared Errors (MSEs) of the selected ridge biasing parameters were assessed as a function of the sample size (n) for fixed values of γ=0.8,0.9,0.95, and 0.99, with p=5 and σ=3,5, and 10. The results indicate a clear inverse relationship between sample size and MSE, demonstrating that estimator efficiency improves as n increases. Across all simulation design, the ridge-type estimators consistently outperformed the Ordinary Least Squares (OLS) estimator, achieving notably smaller MSEs. For smaller sample sizes (e.g., n=50), the Ridge_kgk estimator exhibited superior performance relative to other biasing parameters. Similarly, for larger sample sizes (e.g., n=100), Ridge_kgk maintained its dominance, yielding the minimum MSE among the compared estimators.

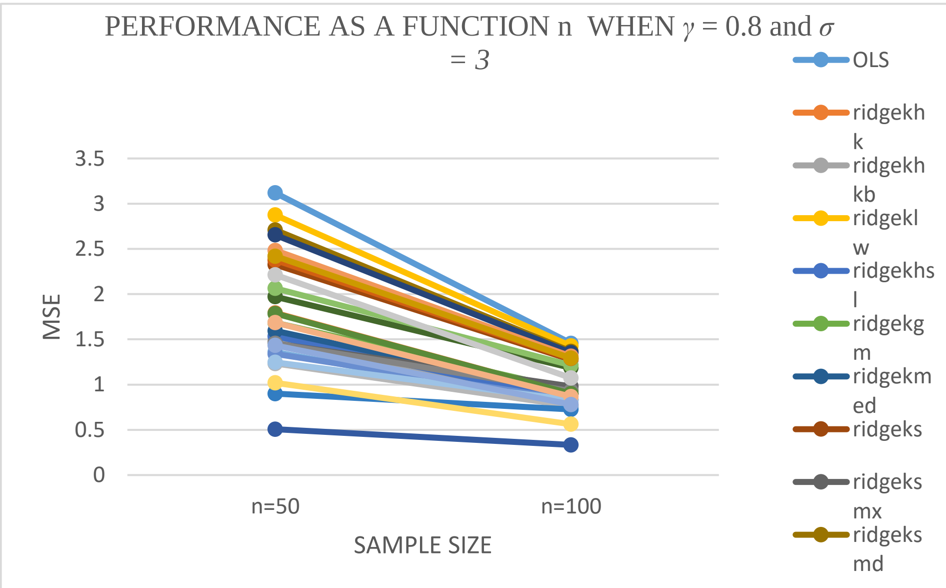

Figure 3: Performance as a function of sample size (n) when *γ* = 0.8 and *σ* = *3*

CONCLUSION

Based on the outcomes of the simulation experiment, several key conclusions can be drawn. First, an increase in the error variance (σ) leads to a corresponding rise in the Mean Squared Error (MSE). Similarly, higher levels of multicollinearity, represented by larger values of the correlation coefficient (γ), also result in higher MSEs. Conversely, as the sample size (n) increases, the MSEs tend to decrease, even under conditions of strong multicollinearity and large error variance. Across all experimental configurations, the ridge-type estimators consistently outperformed the Ordinary Least Squares (OLS) estimator, achieving significantly smaller MSEs. Among the estimators considered, the proposed Ridge_kgk and the Ridge_km9 estimator of Muniz *et al*. (2012) exhibited superior performance, producing the lowest MSEs across most scenarios. Therefore, these estimators are recommended for empirical applications where multicollinearity is a concern. The findings further reinforce the theoretical advantage of incorporating optimally selected biasing parameters to enhance estimator stability and predictive accuracy in linear regression models.

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